THE NET MAP ZOO

This zoo contains input files for some NET maps. Derivations of these input files are often given. These derivations find the input file information by constructing NET map presentations, usually based on finite subdivision rules. In addition, derivations of these finite subdivision rules are sometimes given. These derivations of finite subdivision rules are explained, although briefly. The NET map presentations are not explained. They are given diagrammatically.

The purpose of the rest of this file is to explain these diagrams which determine NET map presentations. This is always done on a single page. The top of the page usually has a finite subdivision rule presentation of the given NET map f. We follow the algorithm in Section 8 of [1] for finding NET map presentations. We next state the steps of the algorithm and comment after the statement of each step.

Step 1. Identify the postcritical set P_2 of f.

The cell complexes which appear are in the complex plane. They have two sizes of vertices, small and large. The large vertices are the postcritical points of f. If there are only three of them, then ∞ is a postcritical point.

Step 2. Identify the set P_1 of four points in $f^{-1}(P_2)$ which are not critical points.

We write $f = h \circ g$, as usual. So g is a Euclidean NET map, and h is a push map homeomorphism. The diagram presents g as a cellular map, correct up to isotopy rel P_1. The set P_1 is the set of black vertices in g's image complex, sometimes together with ∞. Some gray vertices and edges sometimes appear to provide a frame of reference.

Step 3. Construct four disjoint (green) arcs β_1, β_2, β_3, β_4 in S^2 each with one endpoint in P_1 and one endpoint in P_2.

The nontrivial green arcs are drawn on g's image complex. Arrows on them indicate the direction in which h pushes.

Step 4. Construct a simple closed curve in S^2 containing P_1 which meets every β_i in at most its endpoints. Label the points of P_1 with labels 0, λ_1, $\lambda_1 + \lambda_2$, λ_2 in cyclic order around the curve. This curve together with this labeling of these four points determines a topological quadrilateral, Q_1, so that the orientation of the labeled points is counterclockwise relative to Q_1.

This is the 1-skeleton of g's image complex. The edges of Q_1 are labeled α, β, γ, δ. Instead of using 0 as a vertex label, we use $\emptyset$ to distinguish it from the complex number 0.

Step 5. (Optional) Construct $g^{-1}(\partial Q_1)$ up to isotopy rel P_1.

This is the 1-skeleton of g's domain complex. Its edges are labeled with the labels of their images under g.

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Step 6. Let γ_s be a simple closed curve in $S^2 \setminus P_2$ with slope s with respect to Q_1 and its labeling. Compute the slope $\frac{p}{q}$ in reduced form of one component of $g^{-1}(\gamma_0)$, and compute the degree d with which g maps this component to γ_0. Compute the slope $\frac{r}{s}$ in reduced form of one component of $g^{-1}(\gamma_\infty)$, and compute the degree e with which g maps this component to γ_∞.

This can be computed from the description of g as a cellular map. The results appear in the bottom left part of the diagram.

Step 7. Multiply one column of the matrix $\left[\begin{smallmatrix} q/d & s/e \\ p/d & r/e \end{smallmatrix}\right]$ by -1 if necessary so that the result has positive determinant. Compute the inverse A of this matrix.

The numerators and denominators of the slopes in Step 6 are chosen so that A already has positive determinant.

Step 8. Let x be the element of P_1 with label 0, and determine the label b of $g(x)$.

This can be computed from the description of g as a cellular map.

Step 9. Construct line segments α_i in F_1 such that the arcs $p_1 \circ q_1(\alpha_i)$ form a set of four disjoint arcs, each with one endpoint in P_1 and one endpoint in P_2.

This appears in the bottom right part of the diagram. The parallelogram F_1 has corners 0, $2\lambda_1$, λ_2 and $2\lambda_1 + \lambda_2$, where λ_1 and λ_2 are the columns of the matrix A in the bottom left part of the diagram. We circle the vector b which appears in the bottom left part of the diagram. We draw the green line segments using the implicit branched covering map from the plane to the Riemann sphere.

In general, there is more to do, but in all of these examples, we now have an NET map presentation for f.

References

[1] W. J. Floyd, W. R. Parry and K. M. Pilgrim, *Presentations of NET maps*, in preparation.